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Digital Tessellation for Geometry Learning in Primary School: A Quasi-Experimental Study

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Abstract

Aim/Purpose	This study aims to assess the effectiveness of a tessellation-based instructional program supported by digital technologies for enhancing geometric learning in primary school pupils.
Background	Digital education offers various benefits, including increased motivation and engagement, and has been shown to be effective in teaching geometry. Although tessellation activities have been shown to effectively support geometry learning at both secondary and primary school levels, in the current literature, their implementation in digital environments has been explored exclusively in secondary education.
Methodology	A quasi-experimental research design was used, using 3D printers and GeoGebra software with an experimental group and a control group, including pre-tests and post-tests. The students were in Grade 4. The instructional activities were designed according to recent recommendations in geometry education, using real-life contexts, drawing, prediction and imagining shapes, and technological tools.
Contribution	This paper makes two contributions to the field of primary geometry education and educational technology. First, it extends research on the effectiveness of tessellation activities in digital environments for learning geometry from secondary to primary school. Second, it provides new empirical evidence for the effectiveness of digital teaching in enhancing geometric learning outcomes at the primary level. Specifically, the study demonstrates how technological tools like 3D printers and GeoGebra software can be used effectively in primary

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education, positively impacting geometry learning and promoting high levels of student engagement.

Findings
The results indicated that the experimental group showed significant improvements in post-test scores compared to the pre-test, while the control group did not. This supports the hypothesis that a tessellation-based program, implemented digitally, can enhance geometric learning in primary students. Additionally, a questionnaire revealed high satisfaction with the activities, particularly with the use of a 3D printer and GeoGebra software, suggesting that these tools increased student engagement.

Recommendations for Practitioners
Primary school teachers should consider integrating digital tessellation activities into their geometry curriculum. Exploiting digital technologies like GeoGebra software can enhance students' understanding of geometric concepts and increase engagement; moreover, 3D printing can provide tangible outcomes, reinforcing learning through hands-on experiences. Therefore, professional development in these technologies may be necessary.

Recommendations for Researchers
Researchers could aim to isolate the specific effects of educational technologies, particularly 3D printers and GeoGebra software, on geometric learning outcomes. A design with multiple experimental groups would be beneficial: one with tessellation activities using technologies, one with tessellation activities without technologies, one with different activities aimed at achieving the same geometric learning outcomes, and a control group engaged in activities targeting different geometric learning outcomes.

Impact on Society
The findings of this study could have broad implications for primary education and educational technology. They suggest a potential shift in how geometry is taught at the primary level, emphasizing the integration of digital tools and manipulative approaches like tessellation. If teacher training organizations begin to incorporate this program into their curriculum, many educators might adopt it, potentially fostering a better understanding of a fundamental subject like geometry and improving digital literacy in future generations.

Future Research
While digital tessellation activities have been explored in secondary schools, their integration into primary education remains under-researched, highlighting the need for more studies examining their potential to enhance younger students' geometric understanding through digital tools. Additionally, longitudinal studies could explore the long-term impact of digital tessellation activities on students' geometric understanding and spatial skills. Future research could also investigate the transferability of skills acquired through these activities to other areas of mathematics. Comparative studies across different age groups and educational levels could provide insights into the optimal timing for introducing such interventions. Finally, exploring the potential of augmented or virtual reality in tessellation-based geometry learning could open new avenues for research in educational technology and mathematics education.

Keywords
digital teaching, tessellation, primary school, educational technology, quasi-experimental design

INTRODUCTION

This article presents a study analyzing the use of digital technologies in geometry teaching, specifically evaluating the effectiveness of a tessellation-based intervention conducted in a digital environment in enhancing primary school students' geometric learning and engagement. A tessellation-based intervention refers to an instructional approach in which students engage in tessellation activities. Tessellation is the process of forming a pattern of shapes that fit together perfectly without any gaps or overlaps.

This article is structured as follows. First, it presents classical and recent theoretical frameworks on geometry learning, suggesting several teaching practices. Next, the article discusses the role of visuospatial skills in learning geometry, which several researchers consider crucial.

With these theoretical foundations established, the article then reviews existing literature on the use of tessellation in teaching geometry, noting that it is considered a human activity that both involves and enhances geometric knowledge and skills (Faiziyah et al., 2021). Research highlights its benefits in both primary education (Kılıç et al., 2007; Uribe Garzón et al., 2014) and secondary education (Ling & Loh, 2021). Nonetheless, while in secondary schools, tessellation-based activities have been explored in digital environments (Laksmiwati et al., 2023; Morales Ramírez et al., 2021), no studies have been identified that integrate digital teaching methods with tessellation activities for primary school students. This study aims to fill this gap. It tests the hypothesis that a tessellation-based instructional program incorporating digital tools, such as GeoGebra and a 3D printer, can improve geometry learning outcomes while enhancing student motivation and engagement.

The article proceeds to outline the research design used to test this hypothesis, including details on the participants (two 4th-grade classes serving as experimental and control groups), the operational definition and measurement of variables, and the data analysis conducted. Results are presented in terms of the implementation of the instructional program and the differences in pre-test and post-test scores between the two groups, which measure learning outcomes. Additionally, the article discusses the findings of a final questionnaire administered to the experimental group to assess whether the use of 3D printing and GeoGebra software enhanced student engagement and appreciation.

Finally, the results are discussed in light of the theoretical background, recent literature on geometry education (Jablonski & Ludwig, 2023), and existing empirical studies on the use of tessellation in schools. The article concludes with a summary of the study's findings, identifying its contribution to advancing the understanding of tessellation as a tool in digital education, and emphasizing its potential to enrich teaching practices in several significant ways. The study demonstrates how educators can foster a deeper understanding of geometric concepts while increasing student motivation and engagement by integrating digital technologies such as GeoGebra and 3D printing with traditional geometry instruction.

These findings encourage teachers to adopt innovative approaches that combine hands-on, manipulative activities with interactive digital tools, striking a balance between tactile and technological learning experiences. Moreover, the study's insights have broader implications for teacher training programs, which could integrate tessellation-based methodologies into their curricula to better prepare educators for 21st-century classrooms. Such an approach enhances geometry instruction and promotes digital literacy, equipping students with skills essential for navigating the modern world.

THEORETICAL BACKGROUND

LEARNING GEOMETRY: KEY THEORETICAL FRAMEWORKS

A classic reference to the formation of geometric concepts is the work of Piaget and Inhelder (1948), where they distinguish between perceptual space, which children experience through sensory-motor activity, and representational space, which is associated with the intellectual concept of space that children develop through the introduction of language. The latter is constructed and regulated by cognitive processes. Piaget identified three key periods in the development of the concept of space: from birth to 4 months, children exhibit limited visual and motor coordination; from 4 to 12 months, coordination improves, and they begin manipulating objects; and from 13 months to 2 years, active exploration of the environment and language development occurs. During these first two years, perception, sensory activity, motor skills, and representation are crucial as children acquire concepts of proximity, order, inclusion, separation, continuity, and, later, Euclidean forms of space (Piaget & Inhelder, 1948). By age 4, children start to represent topological relationships, while Euclidean and projective spatial relations are understood between 8 and 9 years old. Piaget further identifies three stages in the development of geometric thinking:

1. The first stage focuses on topological concepts, where spatial relationships are examined without considering dimensions or shapes.
2. The second stage introduces the notion of perspective, with the acquisition of concepts like straight lines and right angles.
3. The third stage involves the recognition of two-dimensional and three-dimensional space, utilizing measurement, deductive reasoning, and Euclidean geometry concepts.

As Gioberti (2006) notes, Piaget emphasizes the importance of perceptual activity, motor skills, and representation in the development of sensory-motor intelligence. These elements are fundamental to a teaching approach that uses tessellation to enhance geometry learning. Tessellating the plane with geometric shapes directly engages perception, as it requires an understanding of shapes and their spatial relationships. It also necessitates the ability to compare, transpose, and anticipate shapes to fit them appropriately. By actively engaging in tessellation, children exercise their perceptual skills, improving their ability to perform transformations with geometric shapes and understand their spatial relationships. This can be done by manipulating tangible shapes, performing tessellations on paper, or engaging in digital environments. Active participation in tessellation exercises also encourages children to mentally represent shapes and their relationships, enhancing their ability to imagine and visualize geometric concepts. Finally, when tessellating, children must anticipate which shape is needed to fill a space and solve the problem of how to do so effectively, which can also stimulate their problem-solving skills.

An important theoretical reference for learning geometry is Fischbein's (1993) work, which explores the interaction between concepts and images. Concepts are ideal representations of objects or phenomena, while images are sensory representations. In geometric reasoning, these two aspects merge: geometric figures are concepts and visual images, possessing spatial and conceptual properties simultaneously. Fischbein refers to these as "figural concepts," which combine characteristics of both concepts and figures. These reflect spatial properties such as shape, position, and size as well as conceptual qualities like ideality, abstraction, generality, and perfection. According to Fischbein, geometric thinking develops through the interaction between the visual aspect and formal constraints, culminating in figural concepts. When conceptual and figurative aspects conflict, learning and teaching geometry challenges can arise. Despite this distinction, it is crucial from an educational perspective to harmonize both aspects. Using tessellation in a teaching approach to geometry allows this, as students manipulate physical objects to study their useful properties and then create tessellations that integrate both aspects: concept and image. Furthermore, students are required to create geometric figures in digital environments by recalling conceptual knowledge learned earlier to design figures useful for tessellations.

Duval (1995) proposed the theory of figural apprehension, which focuses on how geometric figures are perceived and understood through different cognitive processes. He argues that recognizing a geometric figure requires applying specific rules to view it geometrically rather than relying on automatic perceptual recognition. To comprehend a figure geometrically, learners must perceive its different dimensions, such as recognizing the cube's 3D structure, 2D faces, 1D sides, and 0D vertices. This process, called dimensional deconstruction, breaks the figure into its figural units across various dimensions. Duval categorizes the cognitive processes involved in comprehending geometric figures into four distinct types:

1. *Perceptual:* This involves interpreting visual information.
2. *Sequential:* The process of breaking down a figure according to its geometric attributes.
3. *Discursive:* Requiring supplementary data to elucidate unclear aspects.
4. *Operative:* Encompassing mental or physical transformations of the figure, such as rotation or subdivision.

Each apprehension plays a unique role in how we perceive, analyze, and interact with geometric shapes. Together, they form a comprehensive framework for understanding how individuals process and work with geometric information. For a figure to function as a heuristic in problem-solving, it must evoke both perceptual apprehension and at least one other type. Duval (1995) also highlights the importance of distinguishing between a physical object and its geometric representation, emphasizing the role of sign systems in developing an understanding of geometric shapes.

Wessels and Van Niekerk (2000) developed the Spatial Operational Capacity (SOC), which emphasizes the importance of providing learners with opportunities to interact with physical and mental objects, fostering skills necessary for developing geometry concepts. The model identifies four key categories of variables that contribute to the complexity of visual images: perception, dimensionality, transformation, and mobility. Perception involves the type of visual stimulus, such as real, virtual, or graphic images. Dimensionality refers to the aspects of objects that learners perceive and process. Transformation focuses on the cognitive processes involved in changing the position or structure of shapes. Mobility addresses whether the stimulus is static or dynamic. Sack and Vazquez (2016) applied the SOC model in a seven-year study in elementary schools, concluding that understanding geometry requires mastering three representational modes: full-scale models (real, manipulable objects), conventional-graphic models (2D representations of 3D objects) and semiotic models (abstract, symbolic representations).

Sfard (2008) developed the commognition theory, which focuses on the role of language in cognitive processes, particularly in the development of geometric concepts. According to Sfard, cognitive and interpersonal communication processes are two aspects of the same phenomenon. To understand how learners develop geometry concepts, it is crucial to study their communicative processes. Effective communication requires meeting expectations based on intentions and avoiding breaches in understanding between speakers and listeners. Sfard's theory analyzes language in terms of discourse, which operates within social and cultural contexts, particularly in mathematics classrooms. For instance, Kaur (2015) used commognition theory to study how 7-8-year-old students developed their understanding of triangles. The discourse followed a progression from visual recognition of objects to informal properties and finally to formal definitions. Each stage involved different routines and vocabulary. Overall, Sfard's (2008) theory provides a framework for examining the role of language in mathematical cognition, emphasizing that communication plays a central role in the development of geometric understanding.

In this perspective, tessellation activities require students to perform both manipulative and abstract operations with physical and mental objects, as suggested by the SOC model of Wessels and Van Niekerk (2000). To promote communication as a tool for developing mathematical concepts (Sfard, 2008), teacher-led discussions and clear linguistic representations of the shapes and objects being manipulated are then required. In this sense, a teaching approach focused on tessellation in primary school allows students to use geometric figures heuristically to solve problems, triggering both perceptual and operational apprehension (Duval, 1995).

THE ROLE OF VISUAL-SPATIAL SKILLS IN GEOMETRY LEARNING

Visual-spatial skills play a crucial role in various learning contexts and occupations, particularly in STEM fields (Flores et al., 2021), such as geometry (Miragliotta et al., 2017). These skills encompass a range of cognitive abilities related to processing and manipulating visual and spatial information, such as spatial perception, visualization, mental rotation, and orientation (Sorby, 1999).

The development of visual-spatial skills occurs gradually during childhood and adolescence. Younger children have a limited understanding of space and spatial relationships between objects, but these abilities develop and improve with age and experience (Vecchi & Cornoldi, 1998). A child's spatial exploration begins with a focus on their own body. After acquiring fundamental motor skills, they use their body as a tool to understand their surrounding environment. They then develop the ability to form internal representations based on spatial data essential for orientation. During this process, they rely on specific reference systems, such as the egocentric system, centered on the first-person perspective, and the allocentric system centered on an external object of reference (Zanatta et al., 2020).

Defining visual-spatial skills solely in terms of spatial orientation or position estimation is limiting. Some theories have attempted to further classify these skills, but it is evident that spatial ability is not a singular concept; it can be subdivided into various components. McGee (1979) distinguishes between visualization, which involves the ability to manipulate and rotate objects, and orientation, which refers to the capacity to maintain spatial orientation in relation to one's body. Linn and Petersen (1985) differentiate:

- *Spatial perception:* the ability to define spatial relationships based on body orientation.
- *Mental rotation:* the ability to rotate two-dimensional and three-dimensional objects.
- *Spatial visualization:* the ability to manipulate spatial information presented in an unconventional format.

More recently, Kimura (2000) identified six key spatial parameters widely recognized in experimental measurements: spatial orientation, spatial location memory, targeting, spatial visualization, disembedding, and spatial perception. According to Tiwari et al. (2024), spatial ability is a multifaceted construct that encompasses several subconstructs, with consensus forming around three primary ones: mental rotation (MR), spatial visualization (SV), and spatial perception (SP) (Cho & Suh, 2022; Linn & Petersen, 1985). Cho and Suh (2022) focused on mental rotation (MR) and spatial visualization (SV), omitting spatial perception due to its reliance on body movement, which makes it harder to measure. They defined mental rotation as the ability to mentally rotate 3D objects and visualize them quickly. Spatial visualization was broken down into three subtypes, validated through extensive research from 2012 to 2021:

- SV I.A (2D-3D) – the ability to convert 2D information into 3D and find the correct viewpoint in relation to the observer's body.
- SV I (2D to 3D) – the ability to interpret 2D drawings (e.g., floor plans) and mentally expand them into 3D forms, exploring various possibilities.
- SV II (3D to 2D) – the ability to compress 3D information into 2D and identify the correct viewpoint by translating volumetric data.

The development of visual-spatial skills is an interconnected process that does not occur in isolation; it is closely linked to other dimensions, such as perception, motor skills, cognition, and emotional-affective aspects. This interconnection contributes to a dynamic balance essential for a child's proper development. Any imbalance between these dimensions can lead to alterations in the perception and recognition of environmental stimuli, increasing the risk of neurodevelopmental disorders (Zanatta et al., 2020).

Several studies highlight the role of visual-spatial skills in geometry learning; for example, Miragliotta et al. (2017) identify seven specific visual-spatial skills and correlate them with corresponding geometric abilities:

1. *Visual organization:* Recognizing figural concepts from incomplete representations
2. *Visual scanning:* Identifying figure properties from representations
3. *Visual reconstructive ability:* Reconstructing figural concepts from various inputs
4. *Image generation ability:* Mentally reproducing figural components
5. *Image manipulation ability:* Using and manipulating figural concept properties
6. *Sequential spatial short-term memory:* Remembering different configurations during manipulation
7. *Long-term spatial memory:* Retaining figural components over time

A study by Fastame (2021) explored the connections between visual-spatial skills and geometry performance in school-age students. The results indicated that students with strong visual-spatial imagination outperformed their peers with lower skills in geometry tasks. Similarly, Lucangeli and Mammarella (2010) highlight that recognizing geometric shapes involves both perception and recalling specific properties from memory.

The ability to manipulate images is also crucial in learning geometry. Research shows that preschool children struggle to recognize congruent figures when they are rotated in space. Until around age six, children can recognize and reproduce figures but have difficulty rotating and transforming mental images (Rosser, 1994). Developmental studies emphasize the importance of actively stimulating children to create and modify mental representations. A metacognitive approach to spatial instruction has been shown to enhance map reading and comprehension skills in children aged four to six (Lucangeli & Mammarella, 2010).

In conclusion, an instructional intervention aimed at acquiring geometric learning in primary school should also focus on developing visual-spatial skills through the manipulation of geometric figures. For example, Miragliotta et al. (2017) suggested enhancing visual-spatial skills through educational interventions using Dynamic Geometry Environments (DGE) and specific geometry software. Their experimental study demonstrated that students improved their ability to identify and classify quadrilaterals based on properties, gained a better conceptual understanding of geometric figures, expanded their knowledge, and improved their construction skills.

Learning Geometry with Digital Teaching

The study by Miragliotta et al. (2017) includes the use of specific geometry software to promote the development of visual-spatial skills and geometric learning. Recently, a systematic review (Sunzuma, 2023) reported that technologies such as GeoGebra and augmented reality have proven effective for teaching and learning geometry.

Digital teaching refers to the integration of digital technology into education to enhance learning experiences and outcomes, fostering skills necessary for digital citizenship and the global economy (Sadiku et al., 2017). Among its benefits are increased accessibility, interactivity, and adaptability in learning (Gu, 2024). Research consistently highlights how digital tools improve student motivation and engagement. For instance, Godzicki et al. (2013) found that technology-supported learning environments positively influence these dimensions in younger students, a finding further supported by Salazar et al. (2019). More recently, Girdzijauskienė et al. (2022) confirmed these benefits in primary education, while tools like 3D printers and design software have been shown to enhance both engagement and learning in various contexts (Kwon, 2017; McGahern et al., 2015; Nikou, 2024).

Regarding its role in teaching geometry, research indicates that digital teaching can be effective in geometry education across various levels. At the early childhood level, educators have successfully integrated digital technologies to engage preschoolers in exploratory geometric learning, though challenges exist (Zhao & Roberts, 2024). In secondary schools, technologies like GeoGebra and augmented reality have shown effectiveness in geometry teaching and learning (Sunzuma, 2023).

Higher education studies demonstrate that digital pedagogy enhances APOS-based learning achievements in advanced geometry courses (Dhakal, 2018). A meta-analysis of studies in Turkey revealed that technology-assisted teaching has a medium effect size on mathematics achievement (0.758) and a very large effect size on geometry achievement (1.136) compared to traditional methods (Çavuş & Deniz, 2022). These findings suggest that digital teaching methods can significantly improve geometry learning outcomes across educational levels, though implementation may vary based on specific contexts and technologies used.

The Role of Dynamic Geometry Software

Franco (2022) highlights the importance of using Dynamic Geometry Software (DGS), which allows for the interactive creation and manipulation of geometric figures. DGSs are specialized computer applications designed to dynamically construct and manage geometric situations. Using specific commands within these software programs – such as point, line, parallel lines, circle, polygon, etc. – users can generate geometric figures that can be easily modified and transformed, thereby facilitating the analysis of properties that remain constant or change during manipulation (Mammana, 2017). Additionally, DGS enables the execution of geometric operations like translations, symmetries, and rotations. The use of computers allows users to achieve an intermediate level of concretization or abstraction between relatively static drawings on paper and the physical manipulation of concrete models (Weigand & Weth, 2002). DGS enhances geometry teaching by offering the ability to construct complex mathematical structures. Their primary advantage over traditional methods lies in their dynamic nature (Dilling & Vogler, 2021).

DGS emerged in the late 1980s, with the first program, Cabri, developed in France by Jean-Marie Laborde and Frank Bellemain, released in 1988 at the International Congress on Mathematical Education (ICME) in Budapest. As the internet spread, numerous similar software programs were developed and gradually adopted in schools, initially in universities and high schools and later in elementary education (Berengo & Terenghi, 2012). The success of these tools in educational institutions is due to their ease of use, the ability to construct and explore Euclidean geometric figures, and the capability to create animations that are difficult to achieve with traditional methods. Using DGS not only enhances geometric understanding but also develops computer skills. Some 3D geometry software allows users to visualize and rotate solid models from different angles (Franco, 2022).

According to Di Paola et al. (2007), DGS encourages hypothesis formation by observing changes resulting from modifications in geometric constructions, fostering a scientific approach to problem-solving. Mammana (2017) notes that teaching with DGS, especially in a lab setting, promotes cooperative learning and the exchange of ideas. Additionally, DGS environments are effective for inclusive education, offering sensory channels that are often more accessible for students with learning difficulties.

Among paid DGS programs, Cabri Geometry, Cinderella, Derive, and the Geometer's Sketchpad stand out. For free options, DrGeo, Gnuplot, and GeoGebra are notable, with GeoGebra being a particularly innovative and widely recognized tool. Created in 2002 by Austrian student Markus Hohenwarter, GeoGebra combines geometry, algebra, graphing, statistics, and calculus into one platform. It is free, open-source, and available under the GNU GPL license, allowing free distribution and use. GeoGebra's widespread success is evident as it has been translated into many languages (Yohannes & Chen, 2023).

Tessellation as a Tool for Learning Geometry

In recent years, various studies have explored the use of two-dimensional tessellation as a tool for teaching and learning geometry. A two-dimensional tessellation is defined as "a division of the plane or a subset of the plane into a collection of disjoint open sets whose closure covers the entire region considered" (Phillips, 2014, p. 202).

From an ethnomathematical perspective, tessellation can be seen as a human activity that involves and promotes geometric knowledge and skills (Faiziyah et al., 2021). Ling and Loh (2021) used plane tessellation with 16-year-old students to enhance cognitive pattern recognition, leading to an improved understanding of geometric and mathematical concepts. In secondary education, Morales Ramírez et al. (2021) studied the use of GeoGebra for tessellation activities with students aged 15-18, concluding that GeoGebra plays an important role in promoting argumentative reasoning and supporting discursive processes in geometric problem-solving. Similarly, Laksmiwati et al. (2023) employed dynamic geometry software for teaching geometry through tessellation, finding that these activities helped students grasp geometric concepts. However, their conclusions are drawn from a study of only three secondary school students, limiting the generalizability of the findings.

In the context of primary education, Kılıç et al. (2007) investigated van Hiele's levels of geometric thinking through tessellation, revealing a connection between mathematical outcomes and levels of geometric reasoning. The study found that providing students with various types of tessellation tasks enhanced their experiences with geometric shapes. This research involved nine fifth-grade students and did not incorporate digital teaching methods.

The most comprehensive educational proposal found in the literature regarding the use of tessellation in primary education is presented by Uribe Garzón et al. (2014). Their study outlines a five-unit curriculum on tessellation aimed at developing spatial thinking skills (such as visual-motor coordination, figure-ground perception, and perceptual constancy) and building geometric knowledge. The curriculum covers Euclidean concepts (lines, vertices, polygons), topological concepts (region, interior, boundary), projective concepts, and plane isometries. It also includes transformations like rotation, translation, and reflection, which are part of transformational geometry. However, the study does not incorporate digital teaching methods, nor does it assess student learning outcomes using an experimental or quasi-experimental design.

Research Question

While previous research has demonstrated the effectiveness of tessellation activities for teaching geometry at both primary and secondary school levels, their integration with digital tools has only been explored at the secondary level. This study aims to address this gap by investigating whether the use of GeoGebra and 3D printing in tessellation-based activities can enhance engagement and improve geometry learning outcomes in primary school students. The instructional approach builds upon key theoretical frameworks in geometry education. Specifically, Duval's (1995) theory of figural apprehension informs the study's focus on how students perceive and manipulate geometric shapes. Fischbein's (1993) concept of figural reasoning supports the integration of digital tools that allow for interactive exploration of geometric properties. The Spatial Operational Capacity (SOC) model (Wessels & Van Niekerk, 2000) underscores the importance of engaging students with both physical and mental manipulations of shapes, while Sfard's (2008) commognition theory highlights the role of discourse in the learning process. By combining these theoretical perspectives with digital tools, this study examines whether a technology-supported approach to tessellation can foster a deeper understanding of geometric concepts in primary education.

Method

Research Design and Hypotheses

To address this research question, a tessellation-based instructional program was designed for primary school students around the age of 9 in Grade 4. To test the hypothesis that this program could positively impact geometry learning outcomes, a quasi-experimental design was employed, specifically a pre-test/post-test design with a control group. Unlike fully experimental designs, this approach does not involve random assignment of participants to groups (Christensen et al., 2011; Cook et al., 1990). Instead, efforts were made to maximize equivalence between the experimental and control groups to strengthen the internal validity of the research design (Kaya, 2015). Two classes from the same school were selected for the study, ensuring they shared the same socio-economic, cultural, and educational context. The school employs a parallel class system, where students across classes of the same grade receive instruction with comparable resources and strategies. Additionally, a pre-test was conducted to confirm that both groups had equivalent geometry learning levels at the start of the study.

The independent variable in this quantitative research design is the tessellation-based instructional program, which was also conducted in a digital environment using GeoGebra software and a 3D printer. The dependent variable is defined by a set of geometry learning outcomes, which are detailed in the following sections.

The study also hypothesized that the integration of digital tools, particularly GeoGebra software and a 3D printer, would enhance student engagement in the experimental group. A qualitative approach was adopted to explore this. A questionnaire with four open-ended questions was administered to the students in the experimental group during the final session of the program to capture their experiences and perceptions. The questions were: (1) What do you think about these activities? (2) Which part did you like the most? (3) Which part did you like the least? (4) Was there any activity in which you had more difficulty? To test the hypothesis, responses to the questionnaire were analyzed for trends.

This multi-method design, combining quantitative and qualitative measures, aimed to provide a comprehensive understanding of the instructional program's impact on both geometry learning outcomes and student engagement.

PARTICIPANTS

The instructional program was implemented in a fourth-grade class at the Galileo Galilei Primary School, part of the Galluzzo School Institute in Florence. In the previous school year, one of the authors of this article had been working as a teacher in this specific class at the school, and she co-designed the educational intervention in collaboration with the class teachers. Therefore, the experimental group in this study was selected because the teacher had a thorough understanding of the group's prerequisites, its relational dynamics, and the suitability of the environment for implementing tessellation activities, considering the available tools, human resources, and materials. The proposal to participate in the study was presented to the class teachers, who readily agreed. Then, this intervention was implemented as a part of regular teaching activities, with tests administered as standard assessment, according to teaching freedom, as stated by Article 33 of the Italian Constitution and exercised within limits established by educational curriculum regulations (Legislative Decree 297/1994; Ministerial Decree 294/2012), teaching autonomy (Presidential Decree 275/1999; Law 107/2015), and privacy regulations (Legislative Decree 196/2003; General Data Protection Regulation - EU Regulation 2016/679). However, given the research purpose of the intervention, the study's objectives and methods were also explained to the teachers of the control group and to the students in both groups. Teachers and students were informed that they could withdraw from the study at any time and that the data would be collected anonymously and processed in aggregate form. Lastly, the project was presented to the school principal, who provided written consent for the study to proceed.

The experimental group was composed of 17 students, including 9 girls and 8 boys. Among the students, there was one with a specific learning disorder, diagnosed with dyslexia and dyscalculia. For the control group, another fourth-grade class from the same primary school was selected. This group consisted of 18 students, including 10 boys and 8 girls, with one student diagnosed with dyslexia and another with physical and motor disabilities. Although extended time was considered for the students with specific learning disabilities during the tests, it was not necessary for either the pre-test or the post-test. The student with physical disabilities in the control group was fully capable of completing the tests without any difficulty. Therefore, as recommended by research guidelines for students with disabilities or special educational needs (Bailey, 2008), the scores of all three students were included in the calculation of the class averages.

Independent Variable: The Instructional Intervention

In this research design, the independent variable is the instructional intervention. For the experimental group, the intervention focuses on tessellation, including activities conducted in a digital environment. In contrast, the control group follows the standard geometry curriculum, specifically covering plane figures and angles.

The educational intervention in the experimental group was designed as a Competence Unit (CU). A CU represents a unified work unit centered on a cohesive training path aimed at developing competencies that can be evaluated, certified, and recognized even outside the acquisition context (Capperucci et al., 2016). The intervention comprises nine sessions (21 hours in total), held twice a week from February 14, 2024, to March 20, 2024.

The first two sessions focus on "plane tessellation: discovering shapes and geometric criteria that allow tessellation." The initial 2.5-hour lesson includes an initial test and activities on plane tessellation using tangible wooden materials, as well as observing and studying the results. The activities are organized into three phases. In the first phase, each student is given a blank sheet of paper along with wooden tiles in various geometric shapes. Students begin by selecting one type of tile and attempting to create a tessellation that fills the sheet. On a separate sheet, they note whether the chosen shape can successfully tessellate the plane. In the second phase, students are encouraged to experiment independently or in small groups with different arrangements of the tiles. For example, they might place a square as if it were a diamond or flip trapezoids upside down compared to their usual orientation. This exploration allows students to investigate the versatility and limitations of different shapes in tessellation. The third phase involves a group discussion to review the results of their experiments. The discussion emphasizes two key "rules" of tessellation: (1) shapes with at least one curved edge cannot be used to tessellate, and (2) tiles suitable for tessellation must fit together seamlessly without gaps or overlaps.

The second 2.5-hour session begins with a brief recap and then aims to facilitate the discovery of the geometric criterion for tessellating a plane: "the sum of internal angles at the vertex is a full rotation." Throughout the activity, students are encouraged to explore and experiment independently or in small groups, identifying which shapes can tessellate the plane and reflecting on the criteria and rules applied.

The third, fourth, and fifth sessions focus on isometries, specifically studying and applying transformations to shapes to create tessellations.

In the first phase of the third session, students are tasked with creating flower-shaped compositions using hexagons, trapezoids, rhombuses, and equilateral triangles. By completing this activity, students observe that the same flower can be made using different quantities of shapes, e.g., 7 hexagons, 14 isosceles trapezoids, 21 rhombuses, or 42 equilateral triangles. This exercise illustrates how tessellations can be modified using various shapes. In the second phase of the third session, students explore irregular shapes that can tessellate a plane. They also examine examples of tessellations found in nature (e.g., reptile shells or scales, bee hives), architecture (e.g., mosaics, walls, and floors), and art, including Escher's works.

In the fourth session, lasting 2 hours and 30 minutes, a video presentation is used to deliver a heuristic lesson on isometries. The concept of translation is introduced first, with examples and characteristics explained in detail. Following this, the concepts of rotation, reflection, and symmetry are presented in sequence. The session concludes with an exploration of combinations of isometries, focusing on cases where a figure undergoes multiple transformations (e.g., symmetry and translation).

The fifth session, lasting 2 hours, focuses on creating a tessellation using translation. To begin, students are shown examples of tessellations on a monitor, after which they are given 5 mm grid paper. They are instructed to draw an irregular tile and test whether it can tessellate the plane when translated in all directions (right, left, up, and down). If the tile successfully tessellates, they fill the entire sheet with the pattern and color the tessellation using two alternating colors.

The sixth session is divided into two groups to provide closer guidance to each student. In the first phase, students are introduced to the GeoGebra software, which is chosen for its simplicity, intuitiveness, and versatility. GeoGebra integrates geometric, algebraic, graphical, statistical, and spreadsheet functionalities in one platform. It is free, open-source (licensed under GNU GPL), and available in multiple languages, including Italian. GeoGebra's international recognition in education further supports its inclusion in this activity. The session's goal is for students to create a tessellation in GeoGebra using translation exclusively, without incorporating rotations or symmetries. First, students create two vectors: one horizontal (three grid spaces to the right) and one vertical (two grid spaces down). They then draw a rectangle with a base of three grid spaces and a height of two grid spaces. Using the "translate" function, the rectangle is tessellated along the vectors, forming a pattern. Students can color each tile and use a checkbox tool to hide or display the tessellation. This allows them to hypothesize the results of manipulating the original figure and then verify their assumptions when reactivating the checkbox. Finally, each student modifies the initial rectangle using the "move" function and the checkbox tool to create an irregular shape suitable for tessellation. The completed shape is then exported in a 3D format, which will be utilized in future sessions.

The final three sessions focus on introducing and using the 3D printer to bring the students' GeoGebra-designed shapes into tangible form. These 3D-printed shapes will be used to create complex physical tessellations.

In the seventh session, lasting 2 hours and 30 minutes, students are introduced to 3D printing technology and observe the printer in action. The session covers various aspects of 3D printing, including its structure, functionality, types of available models, fields of application, common filament materials, software for design, and the machine's setup and adjustment. Students watch the process of creating an object and witness the printing of a tessellation sample from their designs. Due to the time required for printing, which can take several hours, the remaining student-designed tessellations are printed in the days following this session in preparation for the final two sessions.

The eighth session, lasting 2 hours, is dedicated to examining the completed 3D-printed tessellations and using them to recreate the original designs. All printed pieces are mixed and placed on a table, and students are tasked with identifying their individual tiles and reconstructing the tessellation designed in GeoGebra. This activity not only reinforces shape recognition and discrimination but also requires the application of concepts learned throughout the course, such as rotation, translation, and symmetry.

In the final ninth session (2 hours), there is a discussion on each student's work, highlighting learnings and conclusions. A final test on acquired knowledge and a satisfaction questionnaire is administered.

Dependent Variable

The dependent variable in this study is a set of geometry learning outcomes defined operationally through four specific instructional objectives (Mager, 1962):

- Classify plane geometric figures;
- Locate shapes in various positions on the plane;
- Reproduce rotated, translated, and reflected figures;
- Combine irregular shapes to create tessellations.

The selection of these four objectives was guided by various recommendations from the literature. For the first objective, from a theoretical perspective, Duval (1995) highlights the heuristic value of geometric figures, and Sfard (2008) emphasizes the importance of accurate linguistic representation of these figures. In empirical research, Deliyianni et al. (2010) stress the importance of teaching primary school students to classify plane geometric figures, while Bennie and Smit (1999) argue that a primary school curriculum focused on classifying plane figures is essential for preparing students for formal secondary school geometry. For the remaining objectives, the SOC model (Wessels & Van Niekerk, 2000) theoretically recommends providing students with opportunities to interact with both physical and mental objects. Empirically, for the second objective, Rolet (2003) highlights how the ability to locate geometric shapes in different positions on a plane fosters a deeper understanding of geometric objects and their relationships. For the third objective, Yilmaz (2015) stresses the importance of teaching transformational geometry – rotations, translations, and reflections – to develop students' geometric reasoning and visualization skills. Regarding the fourth objective, the tessellation studies analyzed (Kılıç et al., 2007; Laksmiwati et al., 2023; Ling & Loh, 2021; Morales Ramírez et al., 2021; Uribe Garzón et al., 2014) adopted similar objectives.

To measure the achievement of these four objectives, a specially designed test was administered to both the experimental and control groups before and after the intervention. The test includes two questions for each objective, totaling eight questions.

For the instructional objective "classify plane geometric figures," students are required to classify triangles based on sides (isosceles, equilateral, scalene) and angles (acute, obtuse, right) by marking the correct name with an X. The first question allows students to earn between 0 and 8 points if all answers are correct. The second question asks students to identify the correct geometric figure from a set of options based on specific clues (e.g., has four sides, does not have all equal angles, has no right angles, has two pairs of parallel sides) and to write the corresponding letter of the correct figure. This question is scored either 0 or 1.

For the instructional objective "locate shapes in various positions on the plane," the first question requires students to observe several triangles in different positions and identify the two that are identical in shape and size. A correct answer earns 1 point. The second question involves observing a plane figure characterized by an equilateral triangle sharing a side with a square. Students must recognize which other sides are congruent to the one indicated. This question allows for a maximum score of 5 points.

To assess the third instructional objective ("reproduce rotated, translated, and reflected figures"), students must draw the symmetry of an irregular figure in the first question and perform a 180° rotation of a flag in the second. The first question is scored either 0 or 1, while the second is evaluated based on the following criteria: (a) correct 180° rotation, (b) correct placement of the flag on the pole, and (c) accurate distances from the point of rotation. Therefore, the second question has a score range of 0 to 3.

For the fourth instructional objective ("combine irregular shapes to create tessellations"), the first question requires students to identify the two irregular figures missing to complete a tessellation. The second question asks students to observe an incomplete figure missing an irregular piece and to find the shape from the options provided that, when rotated, completes the initial figure. Both questions have a maximum score of 1 point each.

Overall, the test has a maximum raw score of 21 points. The test, translated into English, is included in the appendix.

RESULTS

To identify statistically significant differences between the two groups on the pre-test scores, the normality of the distributions for each group was first verified using the Shapiro-Wilk test. If both distributions were normal, the independent samples t-test would be used to compare the groups; otherwise, the non-parametric Mann-Whitney U test would be applied. To compare pre-test and post-test differences within each group, the normality of the distributions was first checked using the Shapiro-Wilk test. If normal, the paired samples t-test would be used; otherwise, the Wilcoxon signed-rank test would be applied.

Implementation of the Instructional Intervention

The observation process, guided by Corsaro's (1985) notes, allowed for a step-by-step documentation of the instructional intervention. While it is not feasible to include the content of all notes here, this summary highlights the key activities undertaken and any modifications made compared to the original plan outlined in the section dedicated to the instructional intervention.

As initially planned, following the administration of the pre-test, the concept of tessellation was introduced to the experimental group through hands-on exercises involving the use and study of wooden geometric tiles. Subsequent sessions delved deeper into isometries and how they could be employed to create more complex tessellations. The students also engaged in creating tessellations using transformations like translation and explored working with irregular shapes. Later stages introduced the dynamic geometry software GeoGebra, which was used to create tessellations in a digital environment. This allowed students to design irregular shapes suitable for tessellations, which were then printed using a 3D printer after learning about its operation and potential applications. The intervention concluded with the manipulation of the irregular shapes designed and printed by the students.

However, some changes were made during the course of the intervention compared to the original plan. First, in agreement with the classroom teacher, it was decided to separate the pre-test from the first session to optimize time. Second, during the third session, in addition to the planned activities, students were invited to identify tessellations around them within the classroom environment. Third, in the fifth session, instead of practicing isometries on graph paper as initially intended, the students were asked to create a single tessellation using only translation, which would serve as a foundation for future sessions. Lastly, in the seventh session, it was deemed more effective to set up the 3D printer at the beginning of the lesson and then demonstrate its components. This decision was influenced by the time required to print an object, as it was more practical to start the machine early in the lesson to have the printed object ready by the end. In summary, these were minor adjustments that did not alter the content, objectives, or methodological approach of the planned intervention.

Achievement of Instructional Objectives

Before testing the hypothesis through the statistical analyses described in the Data Analysis section, this paragraph examines the post-test results of the experimental group. Specifically, for each of the four objectives outlined in the Dependent Variable section, Table 1 reports the average score achieved by the experimental group, compared with the maximum attainable score, to assess the extent to which each objective was achieved.

Table 1. Descriptive statistics

Instructional objective	Maximum attainable score	Average score achieved	Number of students
Classify plane geometric figures	9	7.47	17
Locate shapes in various positions on the plane	6	4.29	17
Reproduce rotated, translated, and reflected figures	4	2.94	17
Combine irregular shapes to create tessellations	2	1.94	17

As shown in Table 1, the average scores of the experimental group students are very close to the maximum possible scores. This outcome suggests that, overall, the majority of students achieved the four instructional objectives. However, to attribute this result to the educational intervention with a good, though not absolute, degree of validity (Kaya, 2015), it is necessary to test the study hypothesis. This involves considering the control group's results, as well as the pre-test scores of both groups, following the data analysis procedure detailed at the end of the Method section.

HYPOTHESES TESTING

According to the Shapiro-Wilk test, none of the four score distributions (pre-test and post-test for both the experimental and control groups) significantly deviate from a normal distribution (Table 2). As a result, parametric tests can be used. To determine if there is a statistically significant difference between the pre-test scores of the two groups, an independent samples T-test was employed.

Table 2. Shapiro-Wilk test results

Test	Group	W	df	Sign.
Pre-test	Experimental Group (N = 17)	0.905	17	0.083
	Control Group (N = 18)	0.964	18	0.680
Post-test	Experimental Group (N = 17)	0.914	17	0.119
	Control Group (N = 18)	0.945	18	0.356

At the pre-test stage, the experimental group achieved an average score of 13.47 out of a possible 21 points, while the control group scored an average of 13.94 points. The standard deviations were 4.40 points for the experimental group and 3.72 points for the control group (Table 3).

Table 3. Pre/post-test results

Test	Group	Average	Standard deviation
Pre-test	Experimental Group	13.47	4.40
	Control Group	13.94	3.72
Post-test	Experimental Group	16.65	3.64
	Control Group	14.22	3.32

According to the independent samples t-test, the difference between the groups was not significant, $t(33) = -3.34$, $p > .05$. Therefore, the experimental and control groups can be considered equivalent in terms of the level of geometric knowledge, as measured by the pre-test, before the instructional intervention.

Next, the difference between the pre-test and post-test scores for each group was examined using the paired samples t-test. For the control group, the paired samples T-test indicated that the students did not score significantly higher on the post-test (M = 14.22, SD = 3.32) compared to the pre-test (M = 13.94, SD = 3.72), $t(17) = -1.16$, p = .26. However, for the experimental group, the paired samples T-test revealed that the students scored significantly higher on the post-test (M = 16.65, SD = 3.64) compared to the pre-test (M = 13.47, SD = 4.40), $t(16) = -9.19$, $p < .001$. This supports the hypothesis that a tessellation-based instructional intervention, supported by digital technologies, can improve geometric learning in primary school students.

In addition to assessing the impact of the instructional intervention on geometry learning, we sought to evaluate the role of technological tools (the 3D printer and GeoGebra software) in promoting student engagement. Four open-ended questions were posed to the students in the experimental group during the final session. Table 4 summarizes student responses.

Table 4. Number of responses per response category to the open-ended questions in the questionnaire

Open-ended question	Response category	Number of responses
What do you think about these activities?	General appreciation for the program	15
	Appreciation for learning new things	5
	Appreciation for tessellation or using new technologies	3
	Appreciation for using GeoGebra or the 3D printer	3
Which part did you like the most?	The activity involving the 3D printer	17
	The activity with GeoGebra software	4
	The opportunity to manipulate or own the tessellation tiles	2
	The entire instructional program	1
Which part did you like the least?	No part	9
	The test	5
	Specific activities (e.g., drawing the tessellation, the translation part, and a theoretical explanation)	3
	I don’t know	1
Was there any activity in which you had more difficulty?	No difficulties	8
	Difficulties with using the computer or programming	5
	Specific activities (drawing the tessellation, finding the correct shape, completing the tests)	4

The open-ended responses from students reveal a generally positive reception of the instructional program. Many students expressed enthusiasm and enjoyment, often highlighting the program’s engaging and novel aspects, such as learning tessellation techniques and using digital tools like GeoGebra and 3D printers. These elements sparked interest and encouraged creativity, demonstrating the appeal of combining hands-on and technological approaches in geometry education. For example, in response to the question, “What do you think about these activities?” one student wrote: “I think these activities helped me discover new things like tessellation and the 3D printer. I also think they were very creative and fun.”

When asked about their favorite activities, students overwhelmingly gravitated towards the technological components, particularly the 3D printer, which seemed to capture their imagination and foster a sense of ownership through the creation of physical tessellation tiles. For instance, one student wrote: "I liked everything, especially seeing the 3D printer and the objects it produces." GeoGebra also stood out as a tool that enhanced the learning experience, emphasizing the value of integrating dynamic software into classroom activities. Examples of student feedback include: "I really liked when we used GeoGebra" and "My favorite part was programming the shape on the computer."

Although the program was well-received overall, a few students noted challenges, particularly with tasks requiring computer skills or theoretical understanding, suggesting areas where additional support could be provided. For example, one student shared: "The part that gave me some difficulty was programming on the computer." Interestingly, while some disliked traditional elements like testing (e.g., one student wrote: "I didn't like answering the test questions"), others found no aspect of the program unenjoyable, indicating its broad appeal. As one student noted: "I liked everything; there wasn't a part I didn't enjoy."

These findings suggest that the program successfully engaged students by blending interactive, creative, and technological methods, though minor adjustments could further optimize its accessibility and impact.

DISCUSSION, LIMITATIONS, AND CONTRIBUTIONS

DISCUSSION

This study aimed to address the following research question: "Can tessellation-based educational activities conducted in a digital environment enhance engagement and improve geometry learning outcomes among primary school students?" To answer this question, a tessellation-based instructional program was designed for fourth-grade students (approximately 9 years old). The study employed a quasi-experimental design with a control group and no intervention, using dependent samples in a pre-test and post-test format. The experimental group participated in a nine-session instructional intervention focused on tessellation, which included activities in a digital environment using GeoGebra software and a 3D printer. The control group (18 students) followed the standard geometry curriculum.

The results showed that while the control group did not experience significant improvements between the pre-test and post-test, the experimental group achieved significantly higher post-test scores compared to the pre-test. This supports the hypothesis that a tessellation-based instructional intervention implemented in a digital environment can enhance geometric learning among primary school students. The results showed that while the control group did not experience significant improvements between the pre-test and post-test, the experimental group achieved significantly higher post-test scores compared to the pre-test. This supports the hypothesis that a tessellation-based instructional intervention implemented in a digital environment can enhance geometric learning among primary school students.

This study extends the body of research predominantly focused on secondary education to a younger demographic. For example, Ling and Loh (2021) and Morales Ramírez et al. (2021) demonstrated the efficacy of tessellation activities and tools like GeoGebra in fostering cognitive pattern recognition and supporting discursive reasoning among older students. While these studies focused on older students, primarily enhancing pattern recognition and argumentative reasoning, the present study extends these findings to younger learners, showing how digital tools can similarly foster foundational geometric skills. Unlike Laksmiwati et al. (2023), who explored the impact of dynamic geometry software on three students, this study involves a larger sample, providing more robust evidence of the benefits of digital tessellation.

In primary education, previous research by Kılıç et al. (2007) and Uribe Garzón et al. (2014) showed that tessellation activities enhanced geometric reasoning and spatial skills. However, these studies did not incorporate digital methods. This study bridges the gap by demonstrating the effectiveness of digital tools like the 3D printer and GeoGebra in engaging primary students, providing a novel perspective on integrating digital tessellation activities in primary curricula.

Additionally, a questionnaire administered to the experimental group revealed a high level of satisfaction, with most students expressing positive opinions and engagement. The activities involving the 3D printer and GeoGebra software were highlighted as favorites, indicating that using technological tools significantly contributed to student interest and enjoyment. On the other hand, non-technological aspects, such as tests and traditional lessons, were noted as the least enjoyable by some students. These results further reinforce the potential of integrating technology into educational practices to enhance learning experiences.

The success of the instructional program is likely attributable to several factors. First, the approach aimed at integrating elements from both classical and contemporary theories on the development of geometric thinking. In fact, the use of tessellation allowed students to explore geometric concepts through active manipulation, fostering spatial thinking as described by Piaget and Inhelder (1948). It also enhanced both perceptual and operational understanding, as suggested by Duval (1995), and supported the development of mathematical concepts through dialogue and a clear verbal representation, as proposed by Sfard (2008). Additionally, students engaged in both physical and mental manipulation of objects, aligning with the SOC model (Wessels & Van Niekerk, 2000). At the same time, the integration of digital activities facilitated interaction between concepts and images, consistent with Fischbein's (1993) theory of figural concepts. Second, the intervention supported the development of visuospatial skills, which are crucial for learning geometry, as evidenced in studies by Miragliotta et al. (2017) and Fastame (2021). Third, the instructional program included activities consistent with recent recommendations for teaching geometry (Jablonski & Ludwig, 2023). For instance, tessellation examples found in the surrounding environment ensured the real-world connection recommended by Novita et al. (2018); transformations of figures in the plane were explored through drawing, as recommended by Cullen et al. (2018); and the study of isometries activated the imagination of shapes, as recommended by Eilam and Alon (2019). Fourth, the central role of tessellation aligns with research advocating the validity of such activities for enhancing geometric thinking (Faiziyah et al., 2021; Laksmiwati et al., 2023; Ling & Loh, 2021; Morales Ramírez et al., 2021), even at the primary school level (Kılıç et al., 2007; Uribe Garzón et al., 2014). Finally, the use of digital instruction enabled the practice and development of digital skills, as recommended by Dvir and Tabach (2017), and ensured a high level of student engagement and appreciation of the activities, fostering learning, as affirmed by several studies (Girdzijauskienė et al., 2022; Godzicki et al., 2013; Gu, 2024; Sunzuma, 2023), particularly due to the use of the 3D printer and GeoGebra software (Kwon, 2017; McGahern et al., 2015; Nikou, 2024).

Limitations and Future Research

This study has limitations concerning both external and internal validity (Cook et al., 1990). External validity is constrained by the fact that the instructional intervention was conducted with a small group of only 17 students selected as a convenience sample (Christensen et al., 2011). This non-probabilistic approach limits the representativeness of the sample, as its size and selection method do not reflect the broader population of fourth-grade students. Consequently, the study's conclusions are not generalizable to the wider population.

In terms of internal validity, the study is limited by the lack of random assignment of students to the two groups, which would have ensured maximum comparability and, as a result, eliminated the impact of pre-existing significant differences between the groups on the dependent variables. However, as discussed in the research design section, this limitation is mitigated by the fact that these differences are expected to be minimal, given that both classes belong to the same school, share the same socio-economic and cultural context, and show no significant difference in their pre-test scores.

A promising direction for future research would be to replicate this study using an experimental design with random selection and assignment of participants to the experimental and control groups. However, such designs are rare in educational research (Cook et al., 1990) because they typically involve pre-existing school groups that serve as experimental and control groups, making it difficult to establish probabilistic samples representative of the target population (Bailey, 2008). In these conditions, random assignment to experimental or control groups is often not feasible.

Nonetheless, a more feasible study may isolate the effect of the use of technologies (3D printer and GeoGebra software) on the overall geometry learning outcomes selected as the dependent variable. In this study, the research design allows us to assert, with good internal validity (Kaya, 2015), that the tessellation-focused instructional program, which also incorporated digital activities, led to improved geometric learning outcomes among students in the experimental group. However, the design does not allow us to determine whether this improvement is attributable specifically to the educational use of the technologies (3D printer and GeoGebra software). To address this, it would be necessary to replicate the study with a control group experiencing a similar tessellation-focused instructional program but without the digital component. A factorial design with multiple experimental groups would be beneficial: one with tessellation activities using technologies, one with tessellation activities without technologies, one with different activities aimed at the same geometric learning outcomes, and a control group engaged in other activities aimed at different learning outcomes.

Moreover, longitudinal studies could explore the long-term impact of digital tessellation activities on students' geometry understanding and spatial skills. Researchers could investigate the transferability of skills acquired through these activities to other areas of mathematics. Comparative studies across different age groups and educational levels could provide insights into the optimal timing for introducing such interventions. Finally, exploring the potential of augmented or virtual reality in tessellation-based geometry learning could open new avenues for research in educational technology and mathematics education.

CONTRIBUTIONS

Within the limits outlined in the previous section, this study aims to offer a contribution to both practical implementation and theoretical advancement in the field of primary school geometry education and the use of educational technologies. On a practical level, the study demonstrates how the integration of tessellation-based activities, supported by digital tools, can significantly improve various aspects of geometric learning among fourth-grade students. First, the classification of plane geometric figures, according to Duval (1995), serves a heuristic function for solving geometric problems when it involves perceptual apprehension and at least one of the other three apprehensions: sequential, discursive, and operative. Second, tessellation activities, also conducted in digital environments, can enhance learning related to the localization of figures in various positions on a plane, the rotation, translation, and reflection of figures, as well as the combination of shapes to create tessellations. In summary, the benefit for students lies in learning to manipulate and interact with both physical and mental objects and shapes, as suggested by the SOC model of Wessels and Van Niekerk (2000). Furthermore, the use of tools like GeoGebra software and 3D printers has been shown to increase student engagement and improve geometric learning outcomes compared to the control group. This suggests that introducing digital technologies can make geometry instruction more interactive and engaging while providing hands-on learning experiences with tangible tools like 3D printers, fostering learning through practical activities.

From a theoretical perspective, this study fills a gap in the literature regarding the effectiveness of tessellation activities in digital environments, previously explored primarily in secondary education settings (Laksmiwati et al., 2023; Ling & Loh, 2021; Morales Ramírez et al., 2021). In fact, the study demonstrates that younger students can also benefit from the use of digital technologies in tessellation-based activities for geometry learning. It also provides new empirical evidence on the effectiveness of digital instruction in improving geometric learning outcomes in primary education, contributing to a growing body of research on this topic (Çavuş & Deniz, 2022; Dhakal, 2018; Sunzuma, 2023; Zhao & Roberts, 2024). Specifically, the proposed approach highlights how tessellation-based educational activities can combine the use of real-world contexts, drawing, shape imagination, and technological tools, a set of teaching strategies recommended by recent literature on geometry instruction (Jablonski & Ludwig, 2023).

CONCLUSIONS

This study examined the impact of a digital tessellation-based instructional program on primary school students' engagement and geometric learning outcomes. The intervention incorporated digital tools such as GeoGebra and 3D printing to enhance the teaching of tessellations and transformational geometry. A quasi-experimental design was used to compare an experimental group that participated in the digital intervention with a control group following the standard curriculum.

Findings indicate that the experimental group significantly improved their post-test scores compared to the pre-test, while the control group did not show significant changes. This supports the effectiveness of tessellation-based activities in a digital learning environment for enhancing primary students' geometric understanding. Additionally, students in the experimental group reported high engagement, particularly with activities involving the 3D printer and GeoGebra software, reinforcing the role of technology in increasing motivation.

These results align with existing research on digital geometry education at the secondary level, extending the benefits of digital tools to younger students. The study contributes to educational practices by demonstrating how digital tessellation activities can foster both conceptual understanding and hands-on learning. Teachers are encouraged to integrate digital technologies into geometry instruction, balancing them with traditional teaching methods.

Future research should further investigate the specific contributions of digital tools to learning outcomes, possibly through a research design comparing technology-enhanced and non-digital tessellation activities. Longitudinal studies could also explore the long-term effects of digital tessellation activities on students' spatial reasoning and mathematical development. Finally, expanding the research to diverse educational contexts could provide broader insights into the applicability of digital tessellation approaches in primary mathematics education.

REFERENCES

Bailey, K. D. (2008). *Methods of social research*. Simon and Schuster.

Bennie, K., & Smit, S. (1999, July). Spatial sense: Translating curriculum innovation into classroom practice. *5th Annual Congress of the Association for Mathematics Education of South Africa*, *Port Elizabeth, South Africa*, 5-9. https://academic.sun.ac.za/mathed/MALATI/Files/Geometry992.pdf

Berengo, F., & Terenghi, M. (2012). Matematica e tecnologie: Un'interazione dinamica [Mathematics and Technology: A Dynamic Interaction]. *Bricks, 2*(4), 32-46. https://www.rivistabricks.it/wp-content/uploads/2017/08/4_Barengo.pdf

Capperucci, D., Franceschini, G., Guerin, E., & Perticone, G. (2016). *Progettare per unità di competenza nella scuola primaria* [Designing for units of competence in primary school] Franco Angeli.

Çavuş, H., & Deniz, S. (2022). The effect of technology assisted teaching on success in mathematics and geometry: A meta-analysis study. *Participatory Educational Research*, *9*(2), 358-397. https://doi.org/10.17275/per.22.45.9.2

Cho, J. Y., & Suh, J. (2022). The architecture and interior design domain-specific spatial ability test (AISAT): Its validity and reliability. *Journal of Interior Design*, *47*(2), 11-30. https://doi.org/10.1111/joid.12211

Christensen, L. B., Johnson, B., & Turner, L. A. (2011). *Research methods, design, and analysis*. Pearson.

Cook, T. D., Campbell, D. T., & Peracchio, L. (1990). Quasi-experimentation. In M. D. Dunnette, & L. M. Hough (Eds.), *Handbook of industrial organizational psychology* (pp. 491-576). Consulting Psychologists Press.

Corsaro, W. A. (1985). *Friendship and peer culture in the early years*. Ablex.

Cullen, A. L., Eames, C. L., Cullen, C. J., Barrett, J. E., Sarama, J., Clements, D. H., & Van Dine, D. W. (2018). Effects of three interventions on children's spatial structuring and coordination of area units. *Journal for Research in Mathematics Education*, *49*(5), 533-574. https://doi.org/10.5951/jresematheduc.49.5.0533

Deliyianni, E., Elia, I., Gagatsis, A., Monoyiou, A., & Panaoura, A. (2010). A theoretical model of students' geometrical figure understanding. *Proceedings of the 6th Congress of the European Society for Research in Mathematics Education, Lyon, France,* 696-705. https://hal.science/hal-02182374/file/cerme6_proceedings.pdf#page=778

Dhakal, B. P. (2018). Effectiveness of digital pedagogy in higher mathematics education. *The Journal of Development and Administrative Studies*, *26*(1-2), 1-8. https://doi.org/10.3126/jodas.v26i1-2.55607

Dilling, F., & Vogler, A. (2021). Fostering spatial ability through computer-aided design: A case study. *Digital Experiences in Mathematics Education, 7*, 323-336. https://doi.org/10.1007/s40751-021-00084-w

Di Paola, B., Manno, G., Scimone, A., & Sortino, C. (2007). *La Geometria: Una guida ai suoi contenuti e alla sua didattica* [Geometry: A Guide to Its Content and Teaching]. Palumbo.

Duval, R. (1995). Geometrical pictures: Kinds of representation and specific processings. In R. Sutherland, & J. Mason (Eds.), *Exploiting mental imagery with computers in mathematics education* (pp. 142-157). Springer. https://doi.org/10.1007/978-3-642-57771-0_10

Dvir, A., & Tabach, M. (2017). Learning extrema problems using a non-differential approach in a digital dynamic environment: The case of high-track yet low-achievers. *ZDM Mathematics Education*, *49*, 785-798. https://doi.org/10.1007/s11858-017-0862-8

Eilam, B., & Alon, U. (2019). Children's object structure perspective-taking: Training and assessment. *International Journal of Science and Mathematics Education*, *17*(8), 1541-1562. https://doi.org/10.1007/s10763-018-9934-7

Faiziyah, N., Khoirunnia, M., Azizah, N. N., Nurrois, M., & Prayitno, H. J., Desvian, Rustamaji, & Warsito (2021). Ethnomathematics: Mathematics in Batik Solo. *Journal of Physics: Conference Series*, *1720*, 12013. https://doi.org/10.1088/1742-6596/1720/1/012013

Fastame, M. C. (2021). Visuo-spatial mental imagery and geometry skills in school-aged children. *School Psychology International*, *42*(3), 324-337. https://doi.org/10.1177/0143034321992458

Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics*, *24*(2), 139-162. https://doi.org/10.1007/BF01273689

Flores, I., Fallon, C., & Kunda, M. (2021). Visuospatial skills and the workforce: A brief review. *Proceedings of the Annual Meeting of the Cognitive Science Society*, *43*, 3210. https://escholarship.org/uc/item/4422b073

Franco, L. (2022). *Il pensiero geometrico nella scuola primaria: tra realtà e astrazione* [Geometric thinking in primary school: Between reality and abstraction] [Master's dissertation, University of Padua].

Gioberti, R. (2006). Lo sviluppo del concetto di spazio nel bambino vedente e nel non vedente. Una lettura tiflologica di Jean Piaget [The development of the concept of space in sighted and blind children: A tiflological reading of Jean Piaget]. *L'educazione dei minorati della vista. Rivista italiana di tiflologia, 3*, 12-19. https://www.mauriziogalluzzo.it/cladis_0607/forme/pdf/spazio.pdf

Girdzijauskienė, R., Rupšienė, L., & Pranckūnienė, E. (2022). Usage of digital learning tools to engage primary school students in learning. *Proceedings of the 80th International Scientific Conference of the University of Latvia*, *Riga*, 281-290. https://doi.org/10.22364/htqe.2022

Godzicki, L., Godzicki, N., Krofel, M., & Michaels, R. (2013). *Increasing motivation and engagement in elementary and middle school students through technology-supported learning environments* (ED541343). ERIC. https://eric.ed.gov/?id=ED541343

Gu, M. (2024). Empowering vocational education through digital technology: Innovating the teaching landscape. *Institutions*, *6*(6), 131-136. https://doi.org/10.25236/IJNDE.2024.060621

Jablonski, S., & Ludwig, M. (2023). Teaching and learning of geometry – A literature review on current developments in theory and practice. *Education Sciences*, *13*(7), 682. https://doi.org/10.3390/educsci13070682

Kaur, H. (2015). Two aspects of young children's thinking about different types of dynamic triangles: Prototypicality and inclusion. *ZDM Mathematics Education, 47*(3), 407-420. https://doi.org/10.1007/s11858-014-0658-z

Kaya, C. (2015). Internal validity: A must in research designs. *Educational Research and Reviews*, *10*(2), 111-118. https://doi.org/10.5897/ERR2014.1835

Kılıç, Ç., Köse, N. Y., Tanışlı, D., & Özdaş, A. (2007). Determining the fifth grade students' van Hiele geometric thinking levels in tessellation. *Elementary Education Online*, *6*(1), 11-23.

Kimura, D. (2000). *Sex and cognition*. MIT Press. https://doi.org/10.7551/mitpress/6194.001.0001

Kwon, H. (2017). Effects of 3D printing and design software on students' interests, motivation, mathematical and technical skills. *Journal of STEM Education*, *18*(4), 37-42. https://www.learntechlib.org/p/181996/

Laksmiwati, P. A., Hidayah, M., Schmidthaler, E., Prahmana, R. C. I., Sabitzer, B., & Lavicza, Z. (2023). Linking diversity in learning Geometry: Exploring tessellation in techno-based mathematical tasks. *Journal on Mathematics Education*, *14*(3), 585-602. https://doi.org/10.22342/jme.v14i3.pp585-602

Ling, M. K. D., & Loh, S. C. (2021). Relationships between cognitive pattern recognition and specific mathematical domains in mathematics education. *International Journal of Mathematical Education in Science and Technology*, *54*(2), 159-179. https://doi.org/10.1080/0020739X.2021.1949059

Linn, M. C., & Petersen, A. C. (1985). Emergence and characterization of sex differences in spatial ability: A meta-analysis. *Child Development*, *56*(6), 1479-1498. https://doi.org/10.2307/1130467

Lucangeli, D., & Mammarella, I. C. (2010). *Psicologia della cognizione numerica: Approcci teorici, valutazione ed intervento* [Psychology of numerical cognition: Theoretical approaches, assessment, and intervention]. Franco Angeli.

Mager, R. F. (1962). *Preparing instructional objectives*. Fearon.

Mammana, M. F. (2017). Software di Geometria Dinamica nell'insegnamento-apprendimento della geometria: esempi [Dynamic geometry software in the teaching and learning of geometry: Examples]. *Bricks, 2*(4), 114-123.

McGahern, P., Bosch, F., & Poli, D. (2015). Enhancing learning using 3D printing: An alternative to traditional student project methods. *The American Biology Teacher*, *77*(5), 376-377. https://doi.org/10.1525/abt.2015.77.5.9

McGee, M. G. (1979). Human spatial abilities: Psychometric studies and environmental, genetic, hormonal, and neurological influences. *Psychological Bulletin*, *86*(5), 889-918. https://doi.org/10.1037/0033-2909.86.5.889

Miragliotta, E., Baccaglini-Frank, A. E., & Tomasi, L. (2017). Apprendimento della geometria e abilità visuo-spaziali: un possibile quadro teorico e un'esperienza didattica (seconda parte) [Learning Geometry and Visuo-Spatial Skills: A Possible Theoretical Framework and a Teaching Experience (Part Two)]. *L'insegnamento della Matematica e delle Scienze Integrate*, *40*(4), 463-488.

Morales Ramírez, G., Larios Osorio, V., & Rubio Goycochea, N. (2021). Esquemas de argumentación de estudiantes de bachillerato al usar GeoGebra en el contexto de teselados [Argumentation schemes of high school students using GeoGebra in the context of tessellations]. *Uniciencia*, *35*(2), 253-270. https://doi.org/10.15359/ru.35-2.17

Nikou, S. A. (2024). Student motivation and engagement in maker activities under the lens of the Activity Theory: A case study in a primary school. *Journal of Computers in Education*, *11*(2), 347-365. https://doi.org/10.1007/s40692-023-00258-y

Novita, R., Putra, M., Rosayanti, E., & Fitriati, F. (2018). Design learning in mathematics education: Engaging early childhood students in geometrical activities to enhance geometry and spatial reasoning. *Journal of Physics: Conference Series*, *1088*(1), 12016. https://doi.org/10.1088/1742-6596/1088/1/012016

Phillips, D. (2014). Tessellation. *WIREs Computational Statistics*, *6*(3), 202-209. https://doi.org/10.1002/wics.1298

Piaget, J., & Inhelder, B. (1948). *La représentation de l'espace chez l'enfant* [The representation of space in children]. Presses Universitaires de France.

Rolet, C. (2003, February). Teaching and learning plane geometry in primary school: Acquisition of a first geometrical thinking. *Proceedings of the 3rd Conference of the European Society for Research in Mathematics Education, Bellaria, Italy*. http://www.erme.tu-dortmund.de/~erme/CERME3/Groups/TG7/TG7_Rolet_cerme3.pdf

Rosser, R. A. (1994). Children's solution strategies and mental rotation problems: The differential salience of stimulus components. *Child Study Journal, 24*(2), 153-168.

Sack, J., & Vazquez, I. (2016). *A 3D visualization teaching-learning trajectory for elementary grades children*. Springer. https://doi.org/10.1007/978-3-319-29799-6

Sadiku, M. N., Shadare, A. E., & Musa, S. M. (2017). Digital education. *International Journal of Advanced Engineering, Management and Science*, *3*(1), 64-65. https://doi.org/10.53469/jerp.2022.04(12).16

Salazar, J. E. C., Chabla, X. L., Santos, J. P., & Bazán, J. L. T. (2019). Beneficios del uso de herramientas tecnológicas en el aula para generar motivación en los estudiantes [Benefits of using technological tools in the classroom to generate student motivation]. *Revista Ciencias Pedagógicas e Innovación*, *7*(2), 86-93. https://doi.org/10.26423/rcpi.v7i2.304

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press. https://doi.org/10.1017/CBO9780511499944

Sorby, S. A. (1999). Developing 3-D spatial visualization skills. *The Engineering Design Graphics Journal*, *63*(2), 21-32. http://www.edgj.org/index.php/EDGJ/article/view/126/122

Sunzuma, G. (2023). Technology integration in geometry teaching and learning: A systematic review (2010–2022). *LUMAT: International Journal on Math, Science and Technology Education*, *11*(3), 1-18. https://doi.org/10.31129/LUMAT.11.3.1938

Tiwari, S., Shah, B., & Muthiah, A. (2024). A global overview of SVA – Spatial-Visual Ability. *Applied System Innovation*, *7*(3), 48. https://doi.org/10.3390/asi7030048

Uribe Garzón, S. M., Cárdenas Forero, Ó. L., & Becerra Martínez, J. F. (2014). Teselaciones para niños: Una estrategia para el desarrollo del pensamiento geométrico y espacial de los niños [Tessellations for children: A strategy for developing children's geometric and spatial thinking]. *Educación Matemática*, *26*(2), 135-160. https://doi.org/10.24844/EM2602.05

Vecchi, T., & Cornoldi, C. (1998). Differenze individuali e memoria di lavoro visuo-spaziale [Individual differences and visuo-spatial working memory]. *Giornale Italiano di Psicologia*, *25*(3), 491-532. https://doi.org/10.1421/207

Weigand, H. G., & Weth, T. (2002). *Computer in lezioni di matematica* [Computers in mathematics lessons]. Spektrum Akademischer Verlag.

Wessels, D. C., & Van Niekerk, H. M. (2000). Semiotic thinking and the development of secondary school spatial knowledge. *Tydskrif vir Christelike Wetenskap, 3*, 1-16.

Yilmaz, G. K. (2015). The effect of dynamic geometry software and physical manipulatives on candidate teachers' transformational geometry success. *Educational Sciences: Theory and Practice, 15*(5), 1417-1435. https://files.eric.ed.gov/fulltext/EJ1101302.pdf

Yohannes, A., & Chen, H. L. (2023). GeoGebra in mathematics education: A systematic review of journal articles published from 2010 to 2020. *Interactive Learning Environments*, *31*(9), 5682-5697. https://doi.org/10.1080/10494820.2021.2016861

Zanatta, A., Pizzeghello, G., Gasparotto, C., & Battistin, T. (2020). Corpo e mente nello spazio: Le abilità visuo-spaziali [Body and mind in space: Visuo-spatial skills]. *Il TNPEE*, *2*(2), 3-26. https://sfera.unife.it/handle/11392/2480538

Zhao, X., & Roberts, S. (2024). Australian early childhood educators' perspectives on digital teaching of geometry: The pedagogical enablers and barriers. *Australasian Journal of Early Childhood*, *50*(1), 5-18. https://doi.org/10.1177/18369391241234735

LEGISLATIVE REFERENCES

Legislative Decree No. 297, April 16, 1994 - Consolidated Text of Legislative Provisions in the Field of Education, Relating to Schools of Every Order and Degree (Supplement to Gazzetta Ufficiale No.115 of May 19, 1994)

Presidential Decree No. 275, March 8, 1999 - Regulation Containing Rules on the Autonomy of Educational Institutions, Pursuant to Article 21 of Law No. 59, March 15, 1997 (Gazzetta Ufficiale No.104 of May 6, 1999)

Legislative Decree No. 196, June 30, 2003 - Code Regarding the Protection of Personal Data (Gazzetta Ufficiale No.174 of July 29, 2003)

Ministerial Decree No. 294, November 21, 2012 - Regulation Containing Provisions on the Three-Year Programming of Staffing and on the Assignment of Teaching Positions (Gazzetta Ufficiale No.22 of January 26, 2013)

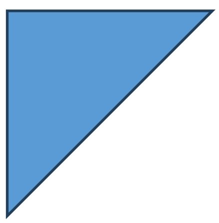
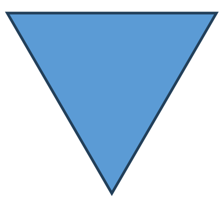
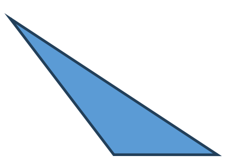
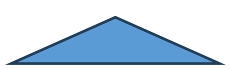
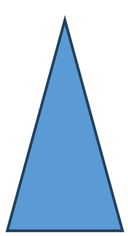
Law No. 107, July 13, 2015 - Reform of the National Education and Training System and Delegation for the Reorganization of Existing Legislative Provisions (Gazzetta Ufficiale No.162 of July 15, 2015)

Regulation (EU) 2016/679 of the European Parliament and of the Council of April 27, 2016 - On the Protection of Natural Persons with Regard to the Processing of Personal Data and on the Free Movement of Such Data (General Data Protection Regulation - GDPR)

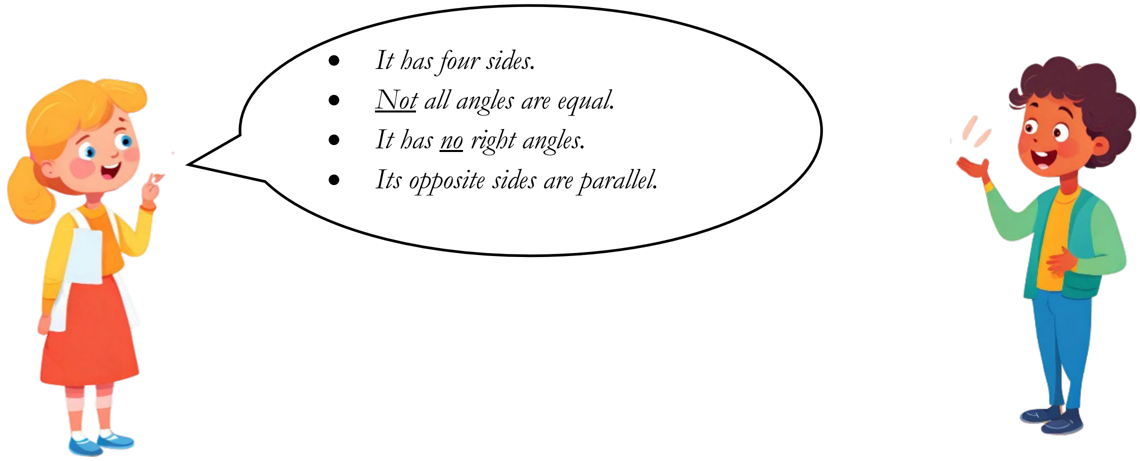
APPENDIX – QUESTIONNAIRE

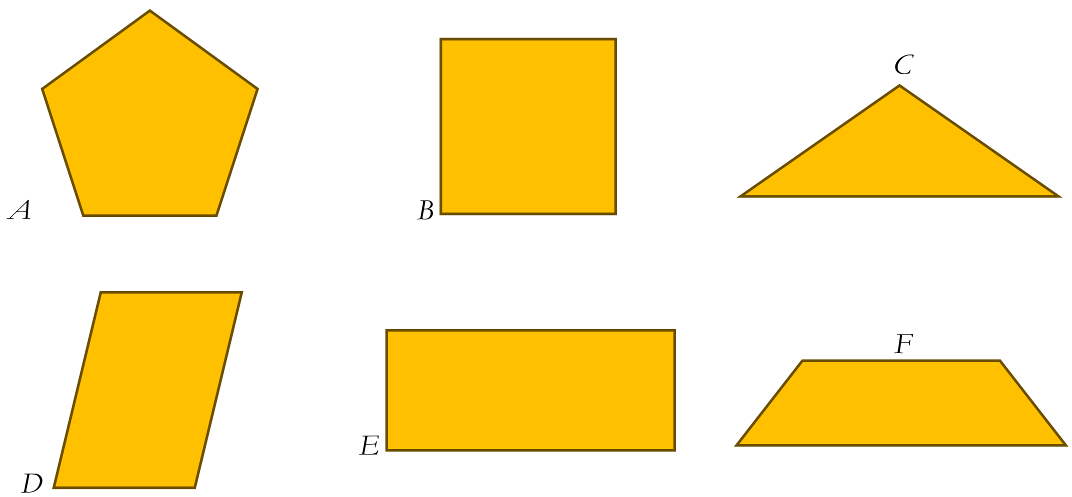
Student Code: ……………………………………………

1. **Observe the following table. Classify each triangle based on its sides and angles, marking the correct columns with an X. Follow the example.**

	Based on sides			Based on angles		
	Isosceles equilateral	Isosceles non-equilateral	Scalene	Right	Obtuse	Acute
		X		X		
						
						
						
						

2. Maria and Luca are playing "Guess the shape."
Follow Maria's clues to figure out what shape it is.

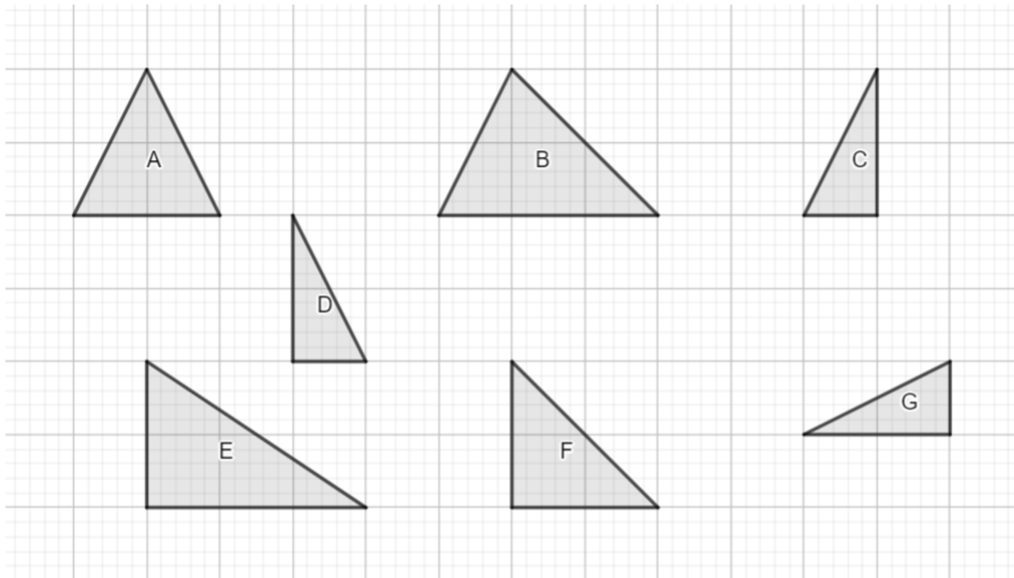




Answer: Maria is describing the figure

……………………………………………………………………………………………

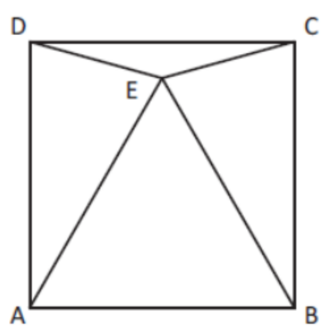
3. **Observe the triangles:**



Which triangles have the same shape and size?

- ☐ Triangles A and B
- ☐ Triangles C, G, and F
- ☐ Triangles E, and F
- ☐ Triangles C, D, and G

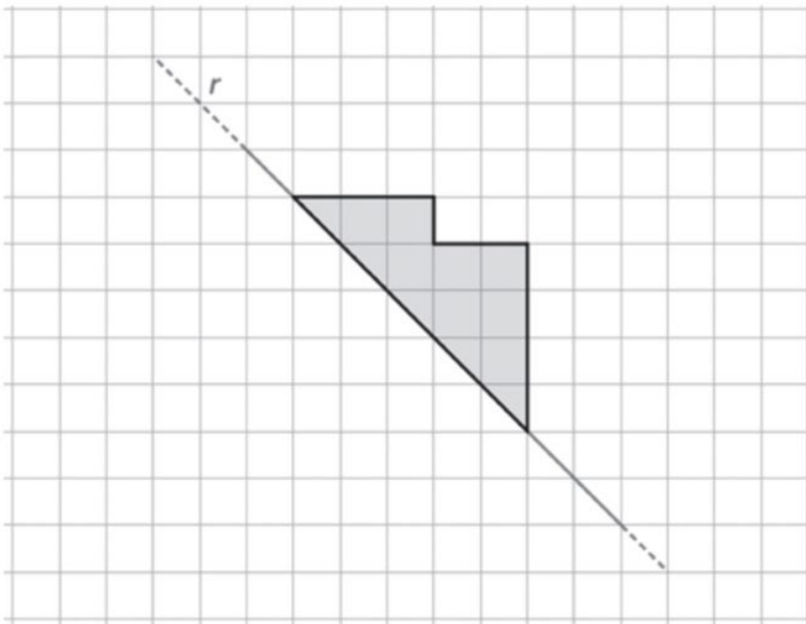
4. **Observe the following flat shape: ABCD is a square, and ABE is an equilateral triangle.**



Which segments have the same length as segment AB?

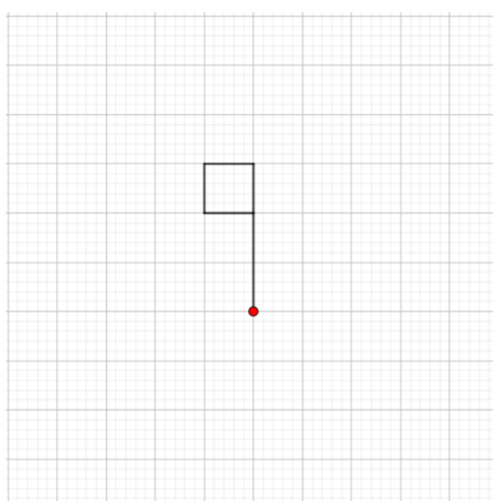
Answer:
....

5. **Observe the gray polygon.**



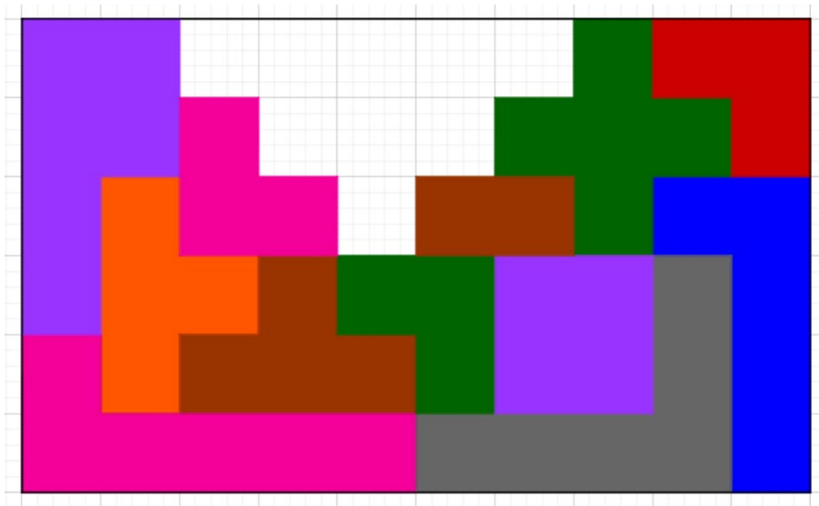
Draw its reflection along axis r.

6. **Draw the figure rotated as instructed:**

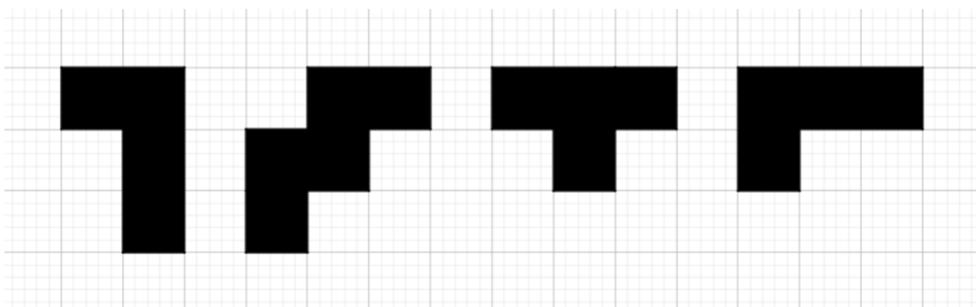


Rotate 180° (½ turn) clockwise around the red point.

7. Observe the following image:



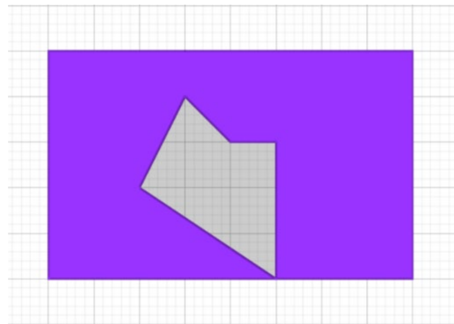
Which of the following shapes are needed to complete the figure?



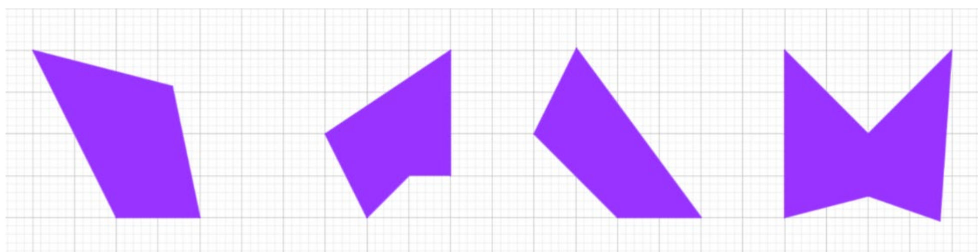
A **B** **C** **D**

- ☐ Shape A and Shape D
- ☐ Shape B and Shape C
- ☐ Shape C and Shape A
- ☐ Shape C and Shape D

8. Observe in the following image the piece missing to complete the purple rectangle:



Which of the following shapes, if rotated, completes the rectangle?



A **B** **C** **D**

☐ Shape A
☐ Shape B
☐ Shape C
☐ Shape D

CREDIT AUTHOR STATEMENT

The contribution reflects the joint effort of all authors; however, Stefano Scippo authored the following sections: "Learning Geometry: Key Theoretical Frameworks," "Learning Geometry with Digital Teaching," "Related Works and Research Question," "Research Design and Hypotheses," "Participants," "Data Analysis," "Achievement of Instructional Objectives," "Hypotheses testing" and "Discussion, Limitations, and Contributions."

Serena Madiai authored the following sections: "The Role of Visual-Spatial Skills in Geometry Learning," "The Role of Dynamic Geometry Software (DGS)," "Independent Variable: The Instructional Intervention," "Dependent Variable," and "Implementation of the Instructional Intervention."

Stefano Cuomo authored the following sections: "Introduction" and "Conclusions."

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